

Name _____

Model Integer Addition

Essential Question How can you use a number line to model addition of integers?

UNLOCK the Problem REAL WORLD

In the first round of a game, Laura lost 5 points. Then she won 9 points in the second round. What is her score after the second round?

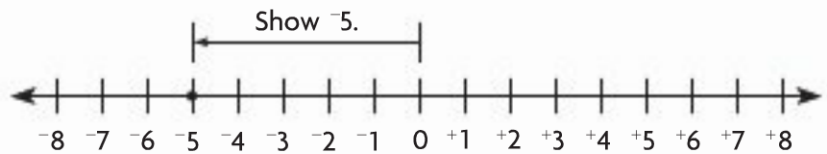
- How can you represent a loss of 5 points with an integer? **-5**
- How can you represent a gain of 9 points with an integer? **+9**

Find $-5 + +9$.

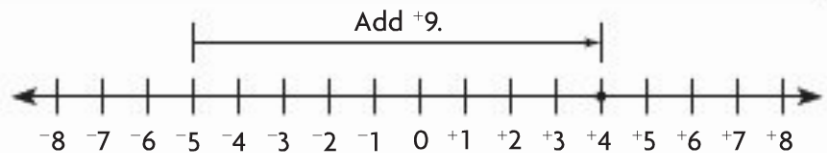
STEP 1 Draw a number line.



STEP 2 Start at 0. Move 5 units to the **left** to show -5 .



STEP 3 From -5 , move 9 units to the **right** to add $+9$.



$-5 + +9 =$ **+4**

So, Laura's score after the second round is **+4**.

Math Talk Will $-5 + +8$ be the same as $+8 + -5$? Explain.

Try This! Tell how to find the sum using a number line.

See below.

A. $+6 + -8$

Start at 0 and move right 6 units. Then move left 8 units. The sum is -2 .

B. $-2 + -6$

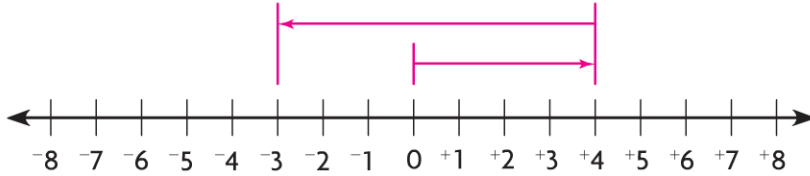
Start at 0 and move left 2 units. Then move left 6 more units. The sum is -8 .

Yes. Possible answer: Moving from zero 5 units left and 8 units right is the same sum as moving from zero 8 units right and 5 units left.

Share and Show



1. Use the number line to find $+4 + -7$.



Start at 0.

Move 4 units to the **right**. Then move 7 units to the **left**.

The sum is -3.

Draw a number line to find the sum.

Check students' drawings.

2. $-3 + +1$ -2

3. $-5 + -1$ -6

4. $+6 + -6$ 0

On Your Own

Draw a number line to find the sum.

Check students' drawings.

5. $-8 + +4$ -4

6. $-3 + -3$ -6

7. $+7 + -9$ -2

8. $+5 + -4$ +1

9. $-4 + -3$ -7

10. $-2 + +10$ +8

Problem Solving

REAL WORLD

11. In a football game, Jim's team gained 7 yards on the first play, lost 2 yards on the second play, and lost 10 yards on the third play. How many total yards did Jim's team gain or lose after three plays?

lost 5 yards

12. In the morning the temperature was -3°F . By noon it had risen by 10°F . What was the temperature at noon?

$+7^{\circ}\text{F}$

Name _____

Model Integer Subtraction

Essential Question How can you use a number line to model subtraction of integers?

UNLOCK the Problem REAL WORLD

At 6:00 P.M., the temperature was -2°F . By midnight, it had dropped 5°F . What was the temperature at midnight?

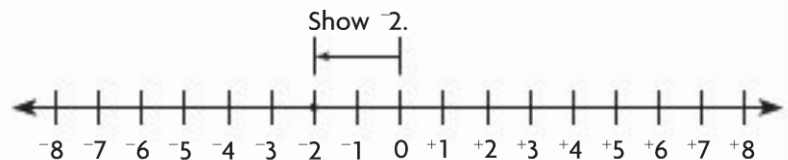
Math Idea

- Move left on a number line to subtract a positive integer.
- Move right on a number line to subtract a negative integer.

Find $-2 - +5$.

STEP 1 Draw a number line.

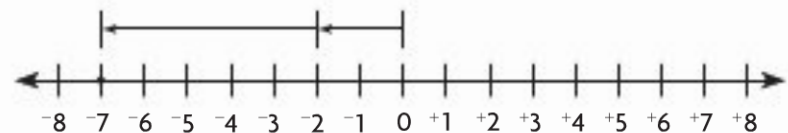
Start at 0, move 2 units to the **left** to show -2 .



STEP 2 From -2 , move 5 units to the

left to subtract $+5$.

$-2 - +5 = \underline{-7}$

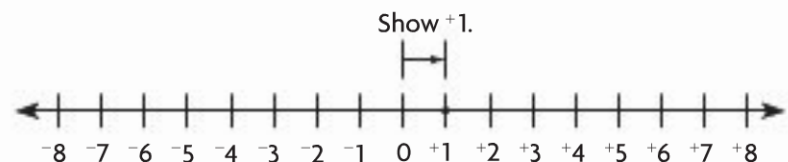


So, the temperature at midnight was -7°F .

EXAMPLE Find $+1 - -4$.

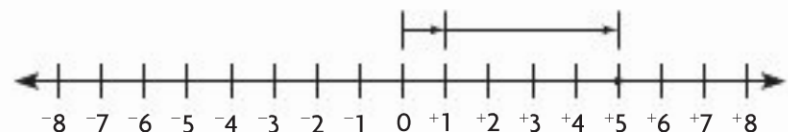
STEP 1 Draw a number line. Start at 0,

move 1 unit to the **right** to show $+1$.



STEP 2 From $+1$, move 4 units to the right to subtract -4 .

So, $+1 - -4 = \underline{+5}$



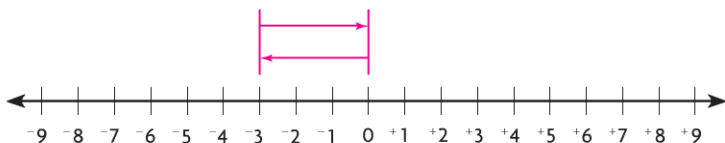
No; Possible answer: $+1 - -4 = +5$. To find $-4 - +1$, first move 4 units left from 0. Then move left 1 more unit to end at -5 .

Math Talk Will $+1 - -4$ be the same as $-4 - +1$? Explain.

Share and Show



1. Use the number line to find $-3 - -3$.



Start at 0.

Move 3 units to the left.

Move right on the number line to subtract -3 .

The difference is 0.

Draw a number line to find the difference. **Check students' drawings.**

2. $-3 - +4$ -7

3. $+5 - +9$ -4

4. $+2 - -4$ $+6$

On Your Own

Draw a number line to find the difference. **Check students' drawings.**

5. $-2 - +2$ -4

6. $+1 - -6$ $+7$

7. $-7 - -7$ 0

8. $+4 - -4$ $+8$

9. $+3 - +6$ -3

10. $-8 - -3$ -5

Problem Solving



11. In a golf tournament, Tim got a score of $+2$ in the first round and a score of -3 in the second round. What was the difference in his scores between the first round and the second round?

$+5$

12. The high temperature one day was -3°F . The low temperature was -7°F . What was the difference between the high and low temperatures that day?

$+4^{\circ}\text{F}$

Name _____

Model Integer Multiplication**Essential Question** How can you use a number line to model multiplication of integers?**UNLOCK the Problem** REAL WORLD

Kayla is scuba diving to explore coral reefs. She makes 5 equal descents of 2 meters each. What is Kayla's elevation at the end of her descent?

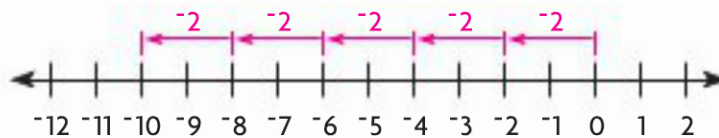
? Find $-2 \times +5$.

By the Commutative Property, $-2 \times 5 = 5 \times -2$.

STEP 1 Draw a number line.



STEP 2 Start at 0. Show five groups of -2 .



$$+5 \times -2 = -10$$

So, Kayla's elevation at the end of her descent is **-10** feet.

Math Idea

5×-2 means 5 groups of -2 .

The sign of the product is negative.

Math Talk

What do you notice about the sign of the product when you multiply a positive integer and a negative integer?

Try This! Tell how to find the product using a number line.

A. $+3 \times (-2)$

Show three groups of -2 . The product is -6 .

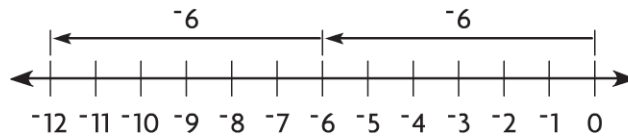
B. $-4 \times +6$

By the Commutative Property, $-4 \times 6 = 6 \times -4$. Show 6 groups of -4 . The product is -24 .

Share and Show



1. Use the number line to find $+2 \times (-6)$.



Start at 0.

Show 2 groups of -6.

The product is -12.

Draw a number line to find the product. **Check students' drawings.**

2. $+4 \times (-1)$

-4

3. $-5 \times +3$

-15

4. $+7 \times (-2)$

-14

On Your Own

Draw a number line to find the product. **Check students' drawings.**

5. $-4 \times +3$

-12

6. $+3 \times (-7)$

-21

7. $-2 \times +4$

-8

8. $+5 \times (-6)$

-30

9. $-9 \times +2$

-18

10. $+6 \times (-8)$

-48

Problem Solving



11. The Milam glacier is changing size at a rate of -3 feet per month. Write a number sentence to show the change in size of the glacier after 3 months.

$3 \times -3 = -9$ feet

12. An elevator starts at the lobby of a building and descends into the basement. The elevator's height changes by -6 meters each second. What is the change in the elevator's height after 3 seconds?

-18 meters

Name _____

Checkpoint

Check Concepts and Skills **Check students' drawings.**

Draw a number line to find the sum, difference, or product. (pp. P261–P262, P263–P264, P265–P266)

1. $+4 + (-2)$

 +2

2. $+5 - (-1)$

 +6

3. $+6 \times (-2)$

 -12

4. $-2 - +7$

 -9

5. $+7 + (-2)$

 +5

6. $-3 \times +1$

 -3

7. $0 - +4$

 -4

8. $+3 \times (-2)$

 -6

9. $-1 + +5$

 +4

10. $-3 \times +7$

 -21

11. $-6 + +8$

 +2

12. $-2 - +4$

 -6

13. $+10 + (-12)$

 -2

14. $-1 - +5$

 -6

15. $-6 \times +3$

 -18

Problem Solving

16. The price of a stock increased by \$6, then decreased by \$8. What integer represents the overall change in the price? (pp. P261)

 -2

17. The captain of a boat is 1 meter above sea level. A diver is 8 meters below sea level. What is the difference in elevation between the captain and the diver? (pp. P263)

 9 meters

18. A researcher lowers the temperature of a specimen by 3° each hour for four hours. What integer shows the total change in temperature? (pp. P261)

 -12

Choose the letter of the correct answer.

19. When Carlos went to bed, the temperature was -2°F . While he slept, the temperature rose by 5°F . What was the temperature when Carlos woke up? (pp. P261–P262)

(A) -7°F

(B) -3°F

(C) 3°F

(D) 7°F

20. To prepare for landing, a jet makes 5 equal descents of 200 meters. Which integer represents the total change in elevation? (pp. P261–P262)

(A) -200

(B) -500

(C) $-1,000$

(D) $-2,000$

21. The temperature at the base of a mountain is 16°F . The temperature at the summit is -4°F . What is the difference in temperatures between the base of the mountain and the summit? (pp. P263–P264)

(A) 20°F

(B) 12°F

(C) -12°F

(D) -20°F

22. In a carnival game, Juan won 11 tickets on his first turn, lost 5 tickets on his second turn, and lost 4 tickets on his third turn. What integer represents the number of tickets Juan has after three turns?

(pp. P265–P266)

(A) 20

(C) 2

(B) -2

(D) -20

23. A submarine sailing 30 meters below sea level rises 20 meters. What integer represents the submarine's new elevation?

(pp. P261–P262)

(A) -50

(C) 10

(B) -10

(D) 20

24. Which of these expressions has the greatest value? (pp. P261–P266)

(A) $-1 + +7$

(C) $+8 - (-3)$

(B) $+2 \times (-4)$

(D) $-4 + +5$

Name _____

Simplify Complex Fractions**Essential Question** How can you simplify complex fractions?A **complex fraction** is a fraction in which the numerator, denominator, or both contain fractions.**UNLOCK the Problem** REAL WORLD

Jerrod swam $\frac{1}{2}$ mile in $\frac{1}{4}$ hour. The complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ is a ratio that represents Jerrod's speed in miles per hour.

? Simplify the complex fraction that shows Jerrod's speed to find the unit rate.

STEP 1 Write the complex fraction as division.

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

STEP 2 Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$$

So, Jerrod's speed was 2 miles per hour.

- What operation does the bar between the numerator and denominator of a fraction represent?

division

Math Talk Explain how you can use division to simplify a complex fraction.

? **Example** Simplify $\frac{\frac{2}{3}}{\frac{4}{5}}$.

STEP 1 Write the complex fraction as division.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5}$$

STEP 2 Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$$

So, $\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{5}{6}$.

Possible answer:
Divide the numerator
by the denominator.

Share and Show



Simplify $\frac{\frac{1}{4}}{\frac{3}{5}}$.

1. Write the complex fraction using division: $\frac{1}{4} \div \frac{3}{5} = \underline{\frac{1}{4} \div \frac{3}{5}}$

2. Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$\frac{1}{4} \times \frac{5}{3} = \underline{\frac{5}{12}}$

Simplify the complex fraction. Write your answer in simplest form.

3. $\frac{\frac{1}{4}}{\frac{2}{3}}$

$\underline{\frac{3}{8}}$

4. $\frac{\frac{4}{5}}{\frac{1}{5}}$

$\underline{4}$

5. $\frac{\frac{1}{2}}{\frac{3}{4}}$

$\underline{\frac{2}{3}}$

On Your Own

Simplify the complex fraction. Write your answer in simplest form.

6. $\frac{\frac{1}{4}}{\frac{2}{5}}$

$\underline{\frac{5}{8}}$

7. $\frac{\frac{1}{6}}{\frac{2}{5}}$

$\underline{\frac{5}{12}}$

8. $\frac{\frac{1}{8}}{\frac{7}{8}}$

$\underline{\frac{1}{7}}$

9. $\frac{\frac{3}{8}}{\frac{3}{4}}$

$\underline{\frac{1}{2}}$

10. $\frac{\frac{2}{9}}{\frac{2}{3}}$

$\underline{\frac{1}{3}}$

11. $\frac{\frac{1}{5}}{\frac{3}{8}}$

$\underline{\frac{8}{15}}$

Problem Solving



12. Meg ran $\frac{7}{8}$ mile in $\frac{1}{8}$ hour. What was her running speed in miles per hour?

$\underline{7 \text{ miles per hour}}$

13. Kareem needs $\frac{3}{4}$ cup of flour to bake a batch of cupcakes. He has $\frac{1}{2}$ cup. What fraction of a batch can Kareem bake?

$\underline{\frac{2}{3} \text{ batch}}$

Name _____

Identify Proportional Relationships**Essential Question** How can you identify a proportional relationship?

A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

UNLOCK the Problem REAL WORLD

Kudzu is a fast-growing plant that is found in the southeastern United States. In summer, kudzu grows 12 inches per day. Is the relationship between the length of a kudzu plant and the number of days it has been growing a proportional relationship?

- What operation can you use to find the length of a kudzu plant after a certain number of days?

multiplication

? Find and compare the ratios of the length of a kudzu plant to the number of days it has been growing.

STEP 1 Make a table of values.

Number of days	1	2	3	4	5
Length (in.)	12	24	36	48	60

STEP 2 Find and compare ratios.

$$\frac{\text{length (in.)}}{\text{number of days}} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = \frac{60}{5} = 12$$

The ratios are constant.

So, the relationship is a proportional relationship.

? **Example**

Judy drives 150 miles in 3 hours, 250 miles in 5 hours, and 400 miles in 8 hours. Is the relationship between distance and time a proportional relationship? If so, what is the unit rate?

Find and compare ratios: $\frac{\text{distance}}{\text{time}} = \frac{150}{3} = \frac{250}{5} = \frac{400}{8} = \underline{50}$

The ratios are constant.

So, the relationship is a proportional relationship.

The unit rate is the ratio that gives the distance traveled in one hour. The unit rate is $\underline{50}$ miles per hour.

Possible explanation: In a proportional relationship, the constant ratio is the unit rate.

Math Talk

Describe the connection between proportional relationships and unit rates.

Share and Show



There are 4 mg of vitamin C in every cup of blueberries. Is the relationship between the amount of vitamin C and the number of cups a proportional relationship?

1. Make a table of values.

Number of cups	1	2	3	4	5
Vitamin C (mg)	4	8	12	16	20

2. Find the ratios of the amount of vitamin C to the number of cups of blueberries.

$$\frac{4}{1} = \frac{8}{2} = \frac{12}{3} = \frac{16}{4} = \frac{20}{5}$$

3. Is the relationship a proportional relationship?

yes

On Your Own

4. Each pound of dried cranberries costs \$3.50. Is the relationship between cost and the number of pounds a proportional relationship?

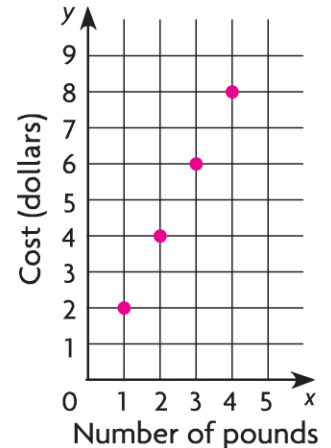
yes

5. The equation $y = 2x$ represents the cost y of buying x pounds of cheese. Complete the table and plot the ordered pairs. Tell whether the relationship between y and x is a proportional relationship and describe what you notice about the points you plotted.

Pounds, x	1	2	3	4
Dollars, y	2	4	6	8

Possible answer: yes; the points lie on a line through the origin.

Cost of Cheese



Problem Solving



6. An elevator rises 40 feet in 2 seconds, 100 feet in 5 seconds, and 180 feet in 9 seconds. Is the relationship between distance and time a proportional relationship? If so, what is the unit rate?

yes; 20 ft/s

7. Drew types 45 words in 1 minute, 120 words in 3 minutes, and 184 words in 4 minutes. Is the relationship between the number of words and time a proportional relationship? If so, what is the unit rate?

no

Name _____

Analyze Proportional Relationships**Essential Question** How can you identify the constant of proportionality in proportional relationships?

A proportional relationship is a relationship between two variables, x and y , that can be written in the form $y = kx$, or $\frac{y}{x} = k$, where k is a nonzero number called the *constant of proportionality*. The graph of a proportional relationship is a straight line through the origin.

UNLOCK the Problem REAL WORLD

Potato salad costs \$3 per pound at a local deli. Write and graph an equation for the proportional relationship. Give the constant of proportionality.

? Analyze the relationship.

STEP 1 Write an equation for the relationship. Let x represent the number of pounds of potato salad. Let y represent the cost of buying x pounds.

Cost = \$3 times the number of pounds

$$y = 3 \cdot x$$

$$y = 3x$$

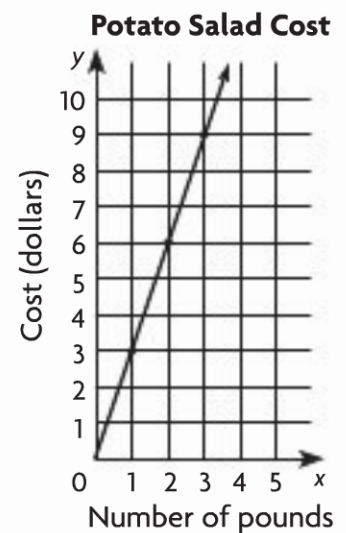
STEP 2 Make a table of values. Then graph the relationship by plotting several points and drawing a line through the points and through the origin.

x	y
1	3
2	6
3	9

STEP 3 Identify the constant of proportionality. The constant of proportionality in $y = 3x$ is 3.

- What operation will you use in your equation for this relationship?

multiplication



It is in the form $y = kx$; the constant of proportionality is 1.

Math Talk Explain why the equation $y = x$ shows a proportional relationship. What is the constant of proportionality?

Share and Show



A shower uses 5 gallons of water per minute. Use this information for 1–3.

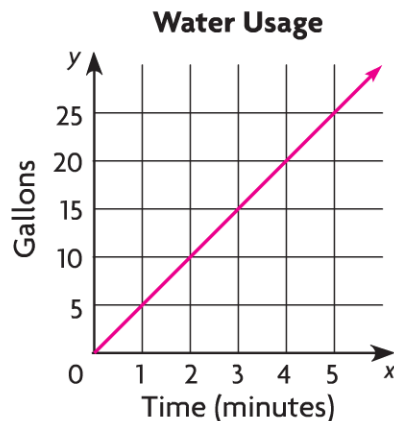
- Let x represent the number of minutes. Let y represent the number of gallons of water used. Write an equation that relates x and y .

$$y = 5x$$

- Give the constant of proportionality.

5

- Graph the equation you wrote in Exercise 1.



On Your Own

Pencils cost \$0.50 each. Use this information for 4–6.

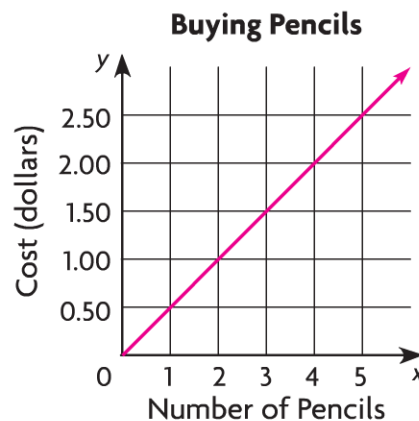
- Let x represent the number of pencils and let y represent the cost. Write an equation that relates x and y .

$$y = 0.50x$$

- Give the constant of proportionality.

0.50

- Graph the equation you wrote in Exercise 4.



Problem Solving



The graph shows the data about a typical whale's heartbeats. Use the graph for 7–9.

- Complete the table.

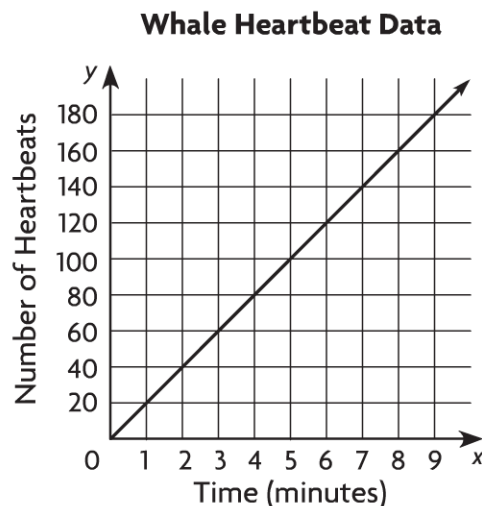
x	1	2	4	7	8
y	20	40	80	140	160

- Use the table to find the ratio $\frac{y}{x}$.

20

- Write an equation that relates x and y .

$$y = 20x$$



Name _____

Apply Percents**Essential Question** How can you solve percent problems involving discounts and sales tax?

A **discount** is a percent of a price that is subtracted from the price. **Sales tax** is a tax that is a percent of a price that is added to the price of an item.

UNLOCK the Problem REAL WORLD

The regular price of a pair of sneakers is \$40. A store is offering a 15% discount on the shoes. What is the sale price?

- How can you write 15% as a decimal?

0.15

Find the discount and sale price of the sneakers.

STEP 1 Find 15% of 40.

$$15\% \text{ of } 40 = \frac{15}{100} \times 40 = 6$$

The discount is \$6.

STEP 2 Subtract to find the sale price.

$$\text{regular price} - \text{discount} = \text{sale price}$$

$$\$40 - \$6 = \$34$$

So, the sale price is \$34.

Example

A pair of jeans costs \$23 plus tax. The sales tax rate is 8%. What is the sales tax? What is the final cost of the jeans?

STEP 1 Find 8% of 23.

$$8\% \text{ of } 23 = \frac{8}{100} \times 23 = \underline{\mathbf{1.84}}$$

The sales tax is \$1.84.

STEP 2 Add to find the total cost.

$$\begin{array}{r} \text{price} + \text{sales tax} = \text{final cost} \\ \$23 + \underline{\mathbf{\$1.84}} = \underline{\mathbf{\$24.84}} \end{array}$$

So, the final cost is **\$24.84**.

10% is \$2.30 and 8% is a bit less than this, so sales tax of \$1.84 is reasonable.

Math Talk How can you use estimation to show that your answer is reasonable?

Share and Show



A video game is on sale for 10% off. The regular price is \$29.00.

1. Find the discount.

$$10\% \text{ of } 29.00 = \frac{10}{100} \times 29 = 2.9$$

The discount is \$2.90.

2. Find the sale price of the game.

$$\begin{array}{r} \text{Regular price} \\ \$29.00 \end{array} - \begin{array}{r} \text{discount} \\ \$2.90 \end{array} = \begin{array}{r} \text{sale price} \\ \$26.10 \end{array}$$

The sale price is \$26.10.

On Your Own

Find the discount and the sale price.

3. regular price: \$50
discount: 20%

$$\begin{array}{r} \text{discount: } \$ \\ \$10 \end{array}$$
$$\begin{array}{r} \text{sale price: } \$ \\ \$40 \end{array}$$

4. regular price: \$56
discount: 25%

$$\begin{array}{r} \text{discount: } \$ \\ \$14 \end{array}$$
$$\begin{array}{r} \text{sale price: } \$ \\ \$42 \end{array}$$

Find the sales tax and the final cost.

5. price: \$75
sales tax: 6%

$$\begin{array}{r} \text{sales tax: } \$ \\ \$4.50 \end{array}$$
$$\begin{array}{r} \text{final cost: } \$ \\ \$79.50 \end{array}$$

6. price: \$25
sales tax: 5%

$$\begin{array}{r} \text{sales tax: } \$ \\ \$1.25 \end{array}$$
$$\begin{array}{r} \text{final cost: } \$ \\ \$26.25 \end{array}$$

Problem Solving



7. A sweater that is regularly sold for \$35 is on sale for 20% off. What is the sale price of the sweater?

\$28

8. Eileen has a \$15 gift card to a music store. She uses the card to pay for a CD that costs \$12 + tax. If the tax rate is 5%, how much will be left on the gift card after the purchase?

\$2.40

Name _____

Percent of Change**Essential Question** How can you find a percent of change?

A **percent of change** is an amount, stated as a percent, that a number goes up or down. If the number goes up, it is a **percent of increase**. If the number goes down, it is a **percent of decrease**. To find a percent of change, use the following formula:

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

UNLOCK the Problem REAL WORLD

The manager of a store raises the price of a pair of shoes from \$40 to \$42. What is the percent of change in the price?

? Use the formula to find the percent of change.

STEP 1 The change is an increase. Find the amount of increase: $42 - 40 = 2$.

STEP 2 Find the percent of increase.

$$\begin{aligned} \text{percent of change} &= \frac{\text{amount of change}}{\text{original amount}} \\ &= \frac{2}{40} \\ &= 0.05 = 5\% \end{aligned}$$

Write the formula.

Substitute.

Divide. Write the quotient as a percent.

So, the percent of change is a 5% increase.

- What clue word tells you that this problem involves a price increase?

“raises”

? **Example** Find the percent of change when the amount of water in a storage tank drops from 640 gallons to 512 gallons.

STEP 1 The change is a decrease. Find the amount of decrease: $640 - 512 = 128$.

STEP 2 Find the percent of change.

$$\begin{aligned} \text{percent of change} &= \frac{\text{amount of change}}{\text{original amount}} \\ &= \frac{128}{640} \\ &= \underline{0.2} = \underline{20\%} \end{aligned}$$

Write the formula.

Substitute.

Divide. Write the quotient as a percent.

So, the percent of change is a 20% decrease.

Possible explanation:
The price doubles.

Math Talk

Explain what it means when a price increases by 100%.

Share and Show



Use these steps to find the percent of change for the prices in the advertisement at right.

Model Train Set
Original Price: \$50
Now reduced to \$29!

1. Tell whether the change is an increase or decrease. Then find the amount of change.

decrease; \$21

2. Substitute values in the formula and divide.

$$\frac{21}{50} = 0.42$$

3. Write the quotient as a percent.

42%

Find the percent of change. Label the change as increase or decrease.

4. 60 is increased to 75.

25% increase

5. 1,200 is decreased to 1,176.

2% decrease

On Your Own

Find the percent of change. Label the change as increase or decrease.

6. 85 is increased to 119.

40% increase

7. 5 is decreased to 4.

20% decrease

8. 35 is decreased to 21.

40% decrease

9. 22 is increased to 44.

100% increase

10. 18 is increased to 26.1.

45% increase

11. 700 is increased to 777.

11% increase

Problem Solving



12. The owner of a boutique buys necklaces from a jewelry maker for \$25 each. Then the boutique owner sells the necklaces for \$40 each. What is the percent of change in the price?

60% increase

13. On Saturday, 400 people attended a school festival. On Sunday, 366 people attended the festival. What is the percent of change in the attendance for the festival from Saturday to Sunday?

8.5% decrease

Name _____

✔ Checkpoint

Check Concepts and Skills

Simplify the complex fraction. Write your answer in simplest form. (pp. P269–P270)

1. $\frac{\frac{1}{4}}{\frac{2}{3}}$ $\frac{3}{8}$

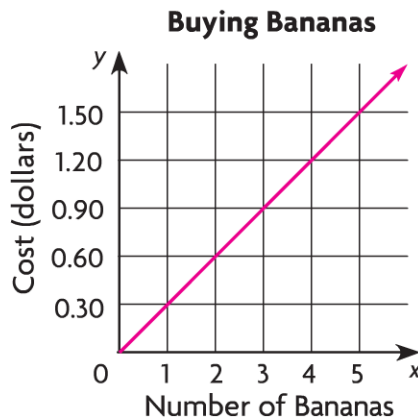
2. $\frac{\frac{1}{6}}{\frac{2}{3}}$ $\frac{1}{4}$

3. $\frac{\frac{3}{5}}{\frac{7}{10}}$ $\frac{6}{7}$

Bananas cost \$0.30 each. Use this information for 4-6. (P271–P274)

4. Let x represent the number of bananas. Let y represent the cost. Write an equation that relates x and y .
5. Give the constant of proportionality.
6. Graph the equation you wrote in Exercise 4.

$$\frac{y = 0.30x}{0.3}$$



Problem Solving REAL WORLD

7. Joelle made 3 bracelets in 15 minutes, 4 bracelets in 20 minutes, and 6 bracelets in 30 minutes. Is the relationship between the number of bracelets and time a proportional relationship? If so, what is the unit rate? (pp. P271–P272) **yes**
8. A calculator that normally sells for \$45 is on sale for 20% off. What is the amount of the discount? (pp. P275–P276)
9. The number of members in the Service Club rose from 18 to 27. What is the percent of change in the membership? (pp. P277–P278)

5 minutes per bracelet

\$9

50% increase

Choose the letter of the correct answer.

10. A box contains $\frac{5}{6}$ pound of cereal. Each serving of cereal is $\frac{1}{12}$ pound. How many servings are in the cereal box? (pp. P269–P270)

- (A) $\frac{5}{72}$ serving (C) 6 servings
 (B) $\frac{5}{2}$ servings 10 servings

11. Kaylin reads 96 pages in 3 hours, 128 pages in 4 hours, and 192 pages in 6 hours. Which statement is correct? (pp. P271–P272)

- (A) The relationship is a proportional relationship. The unit rate is 32 pages per hour.
 (B) The relationship is a proportional relationship. The unit rate is 96 pages per hour.
 (C) The relationship is a proportional relationship. The unit rate is 192 pages per hour.
 (D) The relationship is not a proportional relationship.

12. To attend a school Movie Night, students are asked to donate canned goods. The table below shows data for the relationship between the number of people and the number of cans collected.

People	10	20	40	100
Cans	20	40	80	200

Let x represent the number of people and y represent the number of cans. What is the constant of proportionality for the relationship?

(pp. P273–P274)

- (A) 2 (C) 20
 (B) 10 (D) 100

13. What is the total cost of a cell phone if the price of the cell phone is \$60 and the sales tax rate is 5%? (pp. P275–P276)

- (A) \$60 (C) \$65
 (B) \$63 (D) \$66

14. After exercising, Ellen’s heart rate went from 120 beats per minute to 84 beats per minute. What is the percent of change in her heart rate? (pp. P277–P278)

- (A) 36% decrease (C) 20% decrease
 (B) 30% decrease (D) 16% decrease

Name _____

Add Algebraic Expressions

Essential Question How can you add algebraic expressions?

UNLOCK the Problem REAL WORLD

During track practice, Steven runs laps and then runs cross-country. The expression $3x + 2$ represents the distance in miles he ran on Monday, where x is the length in miles of each lap. The expression $4x + 3$ represents the distance in miles he ran on Tuesday. Write and simplify an expression to show the total distance in miles Steven ran on both days.

Day	Laps	Cross-Country Distance (mi)
Monday	3	2
Tuesday	4	3

? Write and simplify an algebraic expression for the total distance.

STEP 1 Write the algebraic expression for the total distance.

$$\begin{array}{cc} \text{Monday} & \text{Tuesday} \\ \downarrow & \downarrow \\ (3x + 2) & + (4x + 3) \end{array}$$

STEP 2 Use the Associative Property of Addition to remove the parentheses.

$$3x + 2 + 4x + 3$$

STEP 3 Simplify the expression by combining like terms.

Use the Commutative Property of Addition to reorder the terms.

$$3x + 4x + 2 + 3$$

Use parentheses to group like terms.

$$(3x + 4x) + (2 + 3)$$

Combine like terms.

$$7x + 5$$

So, the simplified expression $7x + 5$ represents the total distance in miles. Steven ran $7x + 5$ miles on Monday and Tuesday.

Possible answer: Like terms are terms that have the same variables with the same exponents.

Math Talk

Explain how you can identify like terms in an algebraic expression.

Share and Show



Find the sum of the expressions $(5y + 29)$ and $(8 + 6y)$.

1. Write the algebraic expression for the total.

$$\underline{(5y + 29) + (8 + 6y)}$$

2. Use the Associative Property of Addition to remove the parentheses.

$$\underline{5y + 29 + 8 + 6y}$$

3. Use the Commutative Property of Addition to reorder the terms.

$$\underline{5y + 6y + 29 + 8}$$

4. Use parentheses to group like terms.

$$\underline{(5y + 6y) + (29 + 8)}$$

5. Combine like terms.

$$\underline{11y + 37}$$

Find the sum of the expressions.

6. $(x + 12) + (11 + 2x)$

$$\underline{3x + 23}$$

7. $(4s + 1) + (8s + 14)$

$$\underline{12s + 15}$$

8. $(1 + 41y) + (6y + 2)$

$$\underline{47y + 3}$$

On Your Own

Find the sum of the expressions.

9. $(10t + 7) + (8 + 3t)$

$$\underline{13t + 15}$$

10. $(8 + 3x) + (11 + 17x)$

$$\underline{20x + 19}$$

11. $(9 + 19c) + (18c + 4)$

$$\underline{37c + 13}$$

12. $(16y + 0) + (y + 23)$

$$\underline{17y + 23}$$

13. $(45t + 27) + (23t + 32)$

$$\underline{68t + 59}$$

14. $(35 + 6x) + (6 + x)$

$$\underline{7x + 41}$$

Problem Solving

REAL WORLD

15. Zoe orders 3 books for her friend Amelia and 4 books for her friend Edward. She pays an additional shipping charge of \$5 for Amelia's order and \$7 for Edward's order. Simplify the expression $(3c + 5) + (4c + 7)$, where c represents the cost of each book, to find the total Zoe spent on both orders.

$$\underline{7c + 12}$$

16. Molly works 4 hours on Saturday and earns an additional \$22 in tips. On Sunday, she works 6 hours and earns an additional \$15 in tips. Simplify the expression $(4h + 22) + (6h + 15)$, where h represents the amount she is paid per hour, to find the total she earned over the weekend.

$$\underline{10h + 37}$$

Name _____

Solve Two-Step Equations**Essential Question** How do you solve two-step equations?**UNLOCK the Problem** REAL WORLD

Olivia orders 5 sets of beads. She pays \$7 for shipping, and the total cost of the order is \$52. Solve the equation $5p + 7 = 52$ to find the price p in dollars of each set of beads.

Math Idea

Suppose Olivia had a coupon. What operation would this indicate?

subtraction

? Solve the equation to find the price of each set of beads.

STEP 1 Write the equation.

$$5p + 7 = 52$$

STEP 2 Use the Properties of Equality and inverse operations to get the variable by itself on one side. First undo addition or subtraction, and then undo multiplication or division.

Undo the addition. Subtract 7 from both sides.

$$\begin{aligned} 5p + 7 - 7 &= 52 - 7 \\ 5p &= 45 \end{aligned}$$

Undo the multiplication. Divide both sides by 5.

$$\begin{aligned} \frac{5p}{5} &= \frac{45}{5} \\ p &= 9 \end{aligned}$$

Possible explanation: When I replace p with 9 in the equation, the solution checks: $5 \times 9 + 7 =$ So, the price of each set of beads is \$9. $45 + 7 = 52$.

Math Talk

Explain how you know that your answer is correct.

Try This! Tell how to solve the equation for x .

A. $6x - 9 = 15$

First to undo the subtraction, add 9 to both sides: $6x - 9 + 9 = 15 + 9$; $6x = 24$. Then to undo the multiplication, divide both sides by 6: $\frac{6x}{6} = \frac{24}{6}$; $x = 4$.

Share and Show



1. Solve the equation $\frac{1}{4}c + 6 = 18$.

First undo the **addition** by using **subtraction**.

Then undo the **multiplication** by using **division**.

$c = \underline{\quad 48 \quad}$

Solve the equation.

2. $12x + 2 = 38$

$\underline{\quad x = 3 \quad}$

3. $\frac{1}{3}y - 5 = 3$

$\underline{\quad y = 24 \quad}$

4. $3 + 7p = 52$

$\underline{\quad p = 7 \quad}$

On Your Own

Solve the equation.

5. $23 + 4t = 59$

$\underline{\quad t = 9 \quad}$

6. $2x - 8 = 64$

$\underline{\quad x = 36 \quad}$

7. $5r + 30 = 105$

$\underline{\quad r = 15 \quad}$

8. $\frac{1}{2}p + 15 = 29$

$\underline{\quad p = 28 \quad}$

9. $3c + 58 = 97$

$\underline{\quad c = 13 \quad}$

10. $6y - 37 = 29$

$\underline{\quad y = 11 \quad}$

Problem Solving

REAL WORLD

11. Lee started a round on a game show with 65 points. He answered all 5 questions during the round correctly. Lee's score at the end of the round was 105 points. Solve the equation $65 + 5p = 105$ to find the number of points p that Lee earned for each correct answer.

$\underline{\quad p = 8; 8 \text{ points} \quad}$

12. To repair a bike, a shop charges a fee of \$11, plus \$13 for each hour that the mechanic works on the bike. Minh paid \$63 to have his bike fixed. Solve the equation $11 + 13h = 63$ to find the number of hours h the mechanic worked on Minh's bike.

$\underline{\quad h = 4; 4 \text{ hours} \quad}$

Name _____

Solve Inequalities**Essential Question** How can you solve inequalities?

Solving inequalities is much like solving equations. To solve an inequality, get the variable on one side by itself using the Properties of Inequality and inverse operations.

Addition and subtraction properties of inequality

You can add or subtract the same number on both sides of an inequality, and the inequality will still be true.

$$\begin{aligned} 3 + 2 &< 8 \\ 3 + 2 - 2 &< 8 - 2 \\ 3 + 0 &< 6 \\ 3 &< 6 \end{aligned}$$

Multiplication and division properties of inequality

You can multiply or divide both sides of an inequality by the same positive number, and the inequality will still be true.

$$\begin{aligned} 2 \times 4 &> 6 \\ \frac{2 \times 4}{2} &> \frac{6}{2} \\ 1 \times 4 &> 3 \\ 4 &> 3 \end{aligned}$$

UNLOCK the Problem**REAL WORLD**

A person must be at least 50 inches tall to be allowed to ride a roller coaster. Belinda is 38 inches tall. The inequality $38 + n \geq 50$ can be used to find the number of inches n Belinda must grow to be able to ride the roller coaster. Solve the inequality. Explain what the solution means.

Math Idea

Inequalities may have more than one solution. *Any* value of n that when added to 38 totals more than 50 is a solution for the inequality

$$38 + n \geq 50.$$

**Solve the inequality.****STEP 1** Write the inequality.

$$38 + n \geq 50$$

STEP 2 Use the Properties of Inequality and inverse operations to get the variable by itself on one side.

$$\begin{aligned} 38 - 38 + n &\geq 50 - 38 \\ n &\geq 12 \end{aligned}$$

Undo the addition. Subtract 38 from both sides.

So, the solution of the inequality is $n \geq 12$.

This means that Belinda must grow 12 or more inches before she is able to ride the roller coaster. Any amount of growth she experiences that is 12 inches or more will allow her to ride the roller coaster.

Divide both sides of the inequality by 3.

Math Talk

Describe the inverse operation you would use to solve $3x < 18$.

Share and Show



1. Solve the inequality $2s \leq 6$.

Use the Properties of Inequality and inverse operations to get the variable by itself on one side.

Undo the multiplication by dividing both sides by 2.

The solution is $s \leq 3$.

Solve the inequality.

2. $x + 3 < 4$

$x < 1$

3. $n - 12 > 10$

$n > 22$

4. $\frac{p}{3} \geq 9$

$p \geq 27$

On Your Own

Solve the inequality.

5. $n + 5 < 9$

$n < 4$

6. $x - 1 \leq 0$

$x \leq 1$

7. $7c > 7$

$c > 1$

8. $\frac{m}{2} \geq 2$

$m \geq 4$

9. $a + 16 > 26$

$a > 10$

10. $y - 5 \geq 19$

$y \geq 24$

Problem Solving



11. An elephant weighs more than 30 times what a tiger weighs. An average elephant weighs 12,000 pounds. The inequality $30w < 12,000$ can be used to find the possible weight w in pounds of the tiger. Solve the inequality and explain what the solution means.

$w < 400$; the tiger weighs less than 400 pounds.

12. The inequality $m + 12 \leq 20$ can be used to find the amount of money m in dollars that Nolan can spend at a circus. Solve the inequality and explain what the solution means.

$m \leq 8$; \$8 is the greatest amount Nolan can spend at the circus.

Name _____

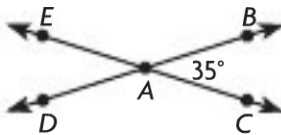
Vertical Angles

Essential Question How can you find the unknown measure of vertical angles?

UNLOCK the Problem REAL WORLD

Vertical angles are formed when two lines or line segments intersect. Vertical angles are opposite congruent angles.

They have the same measure.



- $\angle EAB$ and $\angle DAC$ are vertical angles.
- $\angle BAC$ and $\angle EAD$ are vertical angles.

- What is true about congruent angles?

- What is the measure of a straight angle?

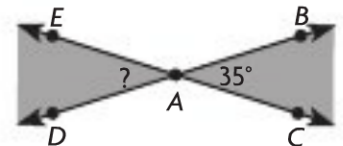
180°

? Find the measure of $\angle EAD$.

THINK: Vertical angles are congruent.

- $\angle BAC$ and $\angle EAD$ are vertical angles, so the measure of $\angle BAC =$ the measure of $\angle EAD$.
- The measure of $\angle BAC$ is 35° .

So, the measure of $\angle EAD$, written $m\angle EAD$, is **35°**.



? Find the measure of $\angle EAB$ and $\angle DAC$.

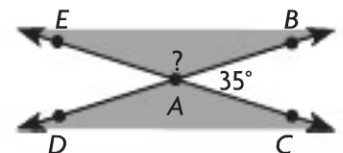
THINK: A straight angle is 180° .

- Together, $\angle EAB$ and $\angle BAC$ make up a straight angle, $\angle EAC$.
- Subtract the measure of $\angle BAC$ from **180°** to find the

measure of $\angle EAB$. $180^\circ - \mathbf{35^\circ} = \mathbf{145^\circ}$

Since $\angle EAB$ and $\angle DAC$ are vertical angles and the measure of

$\angle EAB$ is **145°**, the measure of $\angle DAC$ is **145°**.



Try This!

Materials ■ protractor

- Draw two intersecting lines. Use a protractor to measure one angle.
- Find and label the measure of the other three angles using what you know about vertical angles and straight angles.

Check students' work.

Share and Show

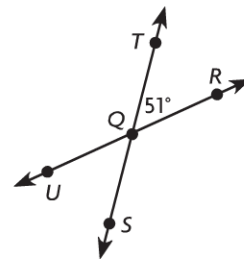


For 1–3, use the drawing to find the measure of the angle.

1. $m\angle SQR = 180^\circ - 51^\circ = \underline{129^\circ}$

2. $m\angle UQS = \underline{51^\circ}$

3. $m\angle UQT = \underline{129^\circ}$



On Your Own

For 4–6, use the drawing to find the measure of the angle.

4. $m\angle PMO =$

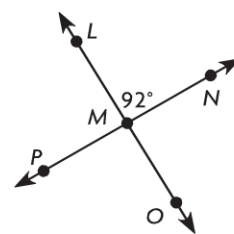
92°

5. $m\angle LMP =$

88°

6. $m\angle NMO =$

88°



For 7–9, use the drawing to find the measure of the angle.

7. $m\angle HIJ =$

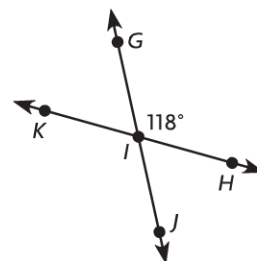
62°

8. $m\angle JIK =$

118°

9. $m\angle KIG =$

62°

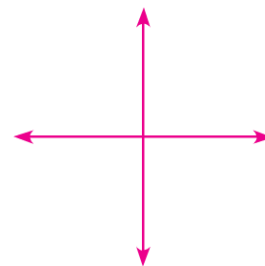


Problem Solving



10. Two intersecting lines form vertical angles that are not obtuse or acute. Describe the angles that are formed and make a sketch of the lines.

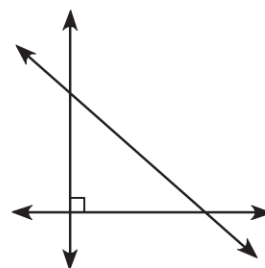
right angles



11. The figure at the right shows three lines intersecting to form an isosceles right triangle. How many acute angles are formed?

Explain why they all are congruent.

4; Possible explanation: The triangle is isosceles, so its acute angles are congruent; the angles opposite these angles are congruent because they are vertical to the triangle's congruent acute angles



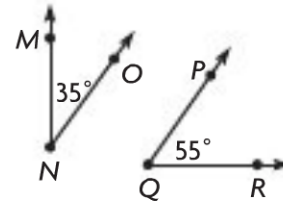
Name _____

Complementary and Supplementary Angles

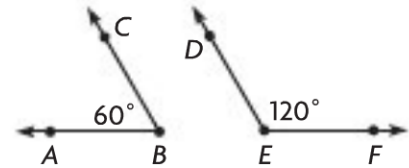
Essential Question How can you find the unknown measure of complementary or supplementary angles?

When the sum of the measures of two angles equals 90° , the angles are **complementary angles**. The angles do not need to be adjacent in order to be complementary. For example, $\angle MNO$ and $\angle PQR$ are complementary, and each angle is the complement of the other.

When the sum of the measures of two angles is 180° , the angles are called **supplementary angles**. You can show that $\angle ABC$ and $\angle DEF$ are supplementary by adding their measures.



$$35^\circ + 55^\circ = 90^\circ$$



$$60^\circ + 120^\circ = 180^\circ$$

UNLOCK the Problem REAL WORLD

The figure shows the support beams of a half-pipe skateboard ramp. Find $m\angle TQG$.

$\angle TOY$ is a right angle, so it measures 90° .

$\angle TQG$ and $\angle GQY$ together form $\angle TOY$, so they are **complementary** angles.

Find the measure of the unknown angle.

STEP 1 The sum of the measures of complementary angles is 90° .

STEP 2 Substitute the measures of the angles.

STEP 3 Solve the equation by using Properties of Equality.

Simplify.

Yes; two 45° angles are congruent and complementary.

So, the $m\angle TQG$ is 35° .



$$m\angle TQG + m\angle GQY = 90^\circ$$

$$x + 55^\circ = 90^\circ$$

$$x + 55^\circ - 55^\circ = 90^\circ - 55^\circ$$

$$x = 35^\circ$$

Math Talk

Explain whether it is possible for two angles to be both congruent and complementary.

Try This! Find $m\angle ABD$.

$\angle ABC$ is a straight angle so it measures 180° , $\angle ABD$ and $\angle CBD$ together form $\angle ABC$, so they are **supplementary** angles.

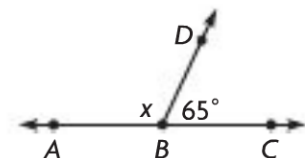
$$m\angle ABD + m\angle CBD = 180^\circ$$

$$x + 65^\circ = 180^\circ$$

$$x + 65^\circ - 65^\circ = 180^\circ - 65^\circ$$

$$x = 115^\circ$$

So, $m\angle ABD = 115^\circ$.



Share and Show



1. Find the unknown angle measure.

STEP 1 The angles are complementary.

$$m\angle PQS + m\angle SQR = 90^\circ$$

STEP 2 Substitute the measures of the angles.

$$71^\circ + x = 90^\circ$$

STEP 3 Solve the equation by using Properties of Equality.

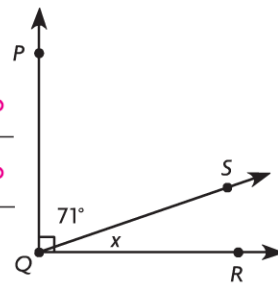
Subtract 71° from both sides.

$$71^\circ - 71^\circ + x = 90^\circ - 71^\circ$$

Simplify.

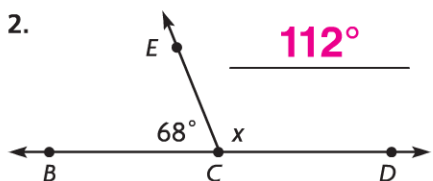
$$x = 19^\circ$$

So, $m\angle SQR$ is 19° .

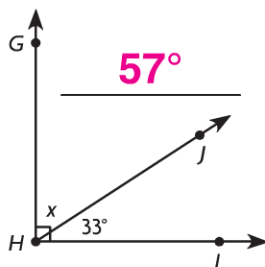


Find the unknown angle measure.

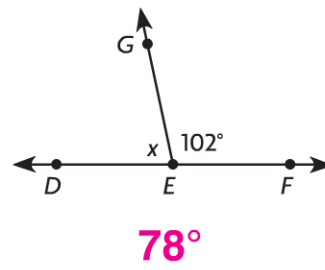
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3.



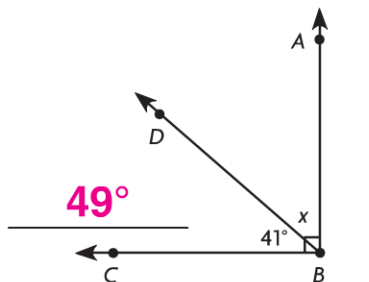
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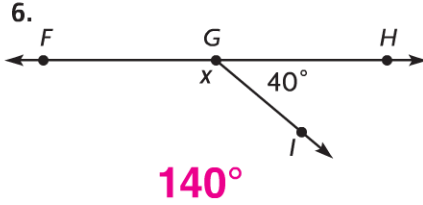
On Your Own

Find the unknown angle measure.

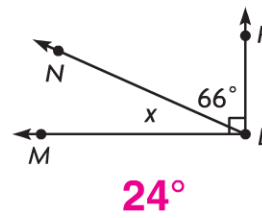
5.



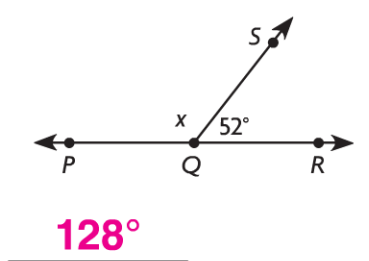
6.



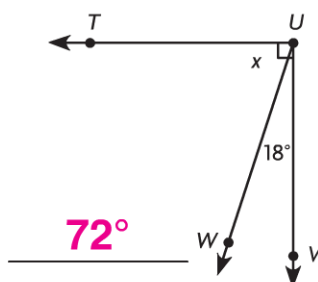
7.



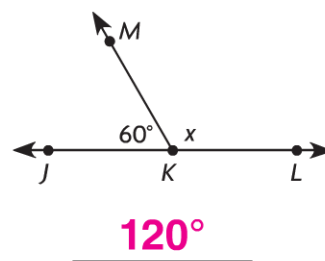
8.



9.



10.

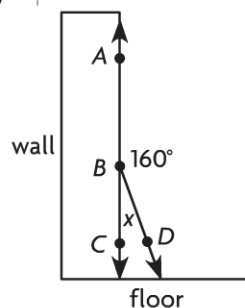


Problem Solving



11. The figure shows a ramp meeting a wall. Find $m\angle CBD$.

$x = 20; m\angle CBD = 20^\circ$



Name _____

Parts of a Circle**Essential Question** How can you identify and draw the parts of a circle?

A **circle** is a closed figure made up of points that are the same distance from a point called the **center**. A circle is named by its center point.

Other parts of a circle include:

- A **radius** is a line segment with one endpoint at the center of the circle and the other endpoint on the circle.
- A **chord** is a line segment that has both of its endpoints on the circle.
- A **diameter** is a chord that passes through the center of the circle.

Use a compass to draw and label the parts of a circle.

Activity**Materials** ■ compass, straightedge**A** Draw circle O with radius \overline{OP} that measures 5 centimeters.

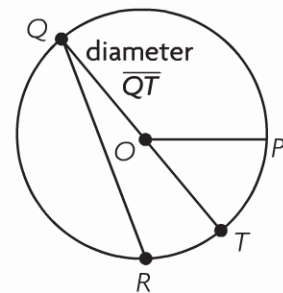
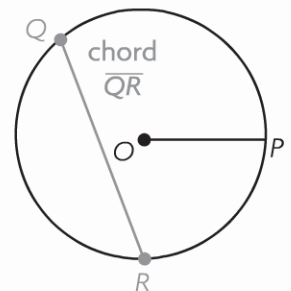
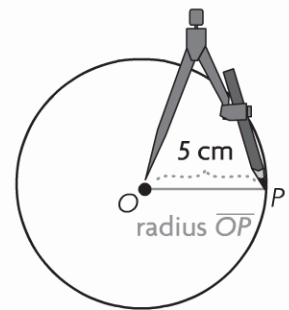
- Draw and label center point O . Place the compass point on it.
- Open the compass to 5 centimeters and draw the circle.
- Label point P on the edge of the circle.
- With a straightedge, draw the radius \overline{OP} .

B Draw chord \overline{QR} on circle O .

- Label points Q and R on the circle.
- Use a straightedge to connect Q and R to create chord \overline{QR} .

C Draw diameter \overline{QT} on circle O .

- Draw a line segment that includes point Q and passes through the center.
- Label point T where the line segment meets the other side of the circle.



The length of the diameter is twice the length of the radius.

Math Talk How is the length of the diameter related to the length of the radius?